A Genetic Algorithm for Topological Characteristics of Kinematic Chains

A genetic algorithm for testing isomorphism among kinematic chains and to select the best frame and input links is presented. The computational effort involved is minimum and the method is unique as it satisfies both the necessary and sufficient requirements. Fitness of a binary string corresponding to a link is indicative of its design parameters. Consequently the fitness of a chain indicates the number of design parameters active in motion generation. Chains are compared for function generation on the basis of the 'fitness' of first generation and second generation 'fitness,' etc., in that order.

Introduction

Generation of kinematic chains has been reported by many investigators. All the methods need tests for isomorphism in order to isolate distinct kinematic chains. Consequently, a number of methods for testing isomorphism have been developed and significant methods among these have been reviewed and listed in [---[1]] and hence not repeated here. With the belief that kinematic analysis and synthesis should not end up only with the generation of kinematic chains, attempts are made by the author [2,3] to compare the kinematic chains and inversions for their anticipated behavior such as mechanical advantage, dynamics, work space and rigidity, etc.

John Holland [4] with his colleagues and students developed genetic algorithms which consider the process of competition, reproduction and the struggle for survival. David Goldberg has made major contributions in this area and in an introductory chapter [5] genetic algorithms with a group of four 5-bit binary strings were illustrated. The works [4,5] come under the study of evolution of intelligence. In this context, the work of Geraldo P. Roster [6] which deals with the hazards in design methodologies, evolutionary algorithms in engineering is noteworthy.

However, with the increasing applications of neural networks in engineering and realizing its potential and lack of its application in the area of kinematics, an attempt is made in this work to utilize the principles (i) to detect isomorphism among kinematic chains, (ii) to compare the chains from the motion generation point of view and (iii) to select the best ground and input links.

Though the fundamental principles of "genetics" are followed broadly, the author has made liberal interpretations in adapting them.

Each link of a chain is considered to have a "fitness" equal to its connectivity. For every link in the chain, other links are considered to be the environment. Formation of a chain is then viewed as mating of links.

Direct joining of two links is considered to be the first generation mating. Combination of links which are separated by one link with respect to every link in the chain is considered as the second generation mating and so on.

A four-link chain is taken as the basic chain and all other chains, six-link, eight-link chains, etc., with the same degree-offreedom (d.o.f.) are viewed as families with different population. The links of a four-link chain are either directly joined or separated by only one link. Likewise, in a six-link chain, the first links are separated by not more than two links and in an eight-link single d.o.f. chain, this is limited to three links and so on. Thus, it is only necessary to study the six-link chains to the extent of first generation mating and eight-links to second generation mating. Obviously, ten-link chains need investigation only up to third generation mating and so on. The above facts establish that any method based on "mating" concept, to test isomorphism among chains, it is necessary to test isomorphism generation-wise, i.e., first generation, second generation mating, etc., depending upon the number of links, obviously it is adequate to test up to the last generation possible viz. up to the third generation in case of ten-link chains. As will be seen later, the effort involved is minimum compared to any other method reported so far. The notable feature is that unlike other methods which propose tests necessary but not sufficient, this method fulfills both necessary and sufficient requirements making it unique.

Link String. The basic principle followed is to represent a link by a string of binary numbers (0 and 1), the number of digits in the string being equal to the number of links in the kinematic chains. The number of times the 1 occurs in the string is equal to the connectivity of the link, i.e., a binary link will have 1 twice in the string while a ternary link will have 1 at three digit places in the string. All other digits in the string are zeros. The exact position of 1 (ones) and 0 (zeros) in the string will depend upon its connection to the other links and their labeling. An important fact to be noted is the "fitness" of a link in the present work is taken to be equal to the connectivity of the link since it is indicative of the number of design parameters possessed by the link. It will also be evident that the fitness of a link is equal to sum of all the nonzero elements in the string.

First Generation Binary Strings of Links

First generation deals with the links that are directly joined. Consider a four-link chain, Fig. 1. It is easy to see that it is the basic chain with least number of links having 1 degree-of-freedom (d.o.f.).

A matrix A, called adjacency matrix, can be written for every chain in which the element

\[ a_{ij} = \begin{cases} 0, & \text{if there is no direct contract between links } i \text{ and } j; \\ 1, & \text{if there is direct contract between links } i \text{ and } j \end{cases} \]

also

\[ a_{ij} = 0, \text{ since a link cannot connect to itself.} \]

For the four-link chain, Fig. 1

\[
\begin{array}{cccc}
\text{Links} & 1 & 2 & 3 \\
1 & 0 & 1 & 0 \\
2 & 1 & 0 & 1 \\
3 & 0 & 1 & 0 \\
4 & 1 & 0 & 1 \\
\end{array}
\]

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Each row of the $A$ matrix can be considered as a binary string of the concerned link with well defined digital position for 0s and is depending upon its connectivity with the other links, i.e., the position of 1 in the binary string is dictated by the environment, i.e., disposition of the other links. Also, as stated earlier, the number of times the element 1 occurs in a row is the link’s connectivity or “fitness.” It is easy to see that the “fitness” of a link is indicative of the number of design parameters it possesses, viz., a binary link has one design parameter while a ternary link has three design parameters.

In a similar manner, the $A$ matrix for a six-link Stephenson chain Fig. 2 can be written as follows.

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \quad (2)$$

Each row of the above matrix can be considered as the binary string of the concerned link with well designed digital positions for the elements 0 and 1 preserving the “fitness” of the link. For example, the binary string of ternary link-3 is $010110$. Relabelling of the links does not matter as the relative positions of the links and hence the matrix elements do not change.

**Mating**

Adjacency matrix for every chain can be written as explained earlier. From the adjacency matrix, each link, having regard to its environment, i.e., to its disposition in relation to other links can be assigned a binary string with 0s and 1s taking definite digital positions, the sum of the nonzero elements being equal to the fitness.

The relationship between any two links or the role of the two links taken together can be studied by mating the binary strings of the concerned links. The following rules are followed.

1. Mating only among the bits (elements) occupying the same digital positions in respective binary strings is possible; e.g., Consider two binary strings,
   
   String $A$ - 0 1 0 1 0 1
   String $B$ - 1 0 1 0 0 0

   Mating of the first bit of string $A$ is possible only with the first bit of the string $B$; similarly second bit of string $A$ can mate with the second bit of string $B$ only

2. The mating of the two strings results in a third string (offspring) $C$ of the same order, i.e., same number of digits. It may be noted that mating is not productive among equal bits, i.e., outcome is no off-spring while mating between unlike bits is possible and productive.

   For example,
   
   Bit of string $A$ + Bit of string $B$ = Bit of string $C$
   
   
   $\begin{align*}
   0 + 1 &= 1 \\
   1 + 0 &= 1 \\
   0 + 0 &= 0 \\
   1 + 1 &= 1 \\
   0 + 1 &= 1 \\
   0 + 0 &= 0
   \end{align*}$

   It can be generalized that

   (i) Mating between the bits occupying the same digital positions in both binary strings need only be considered.

   (ii) Mating among equal bits will result in zero bit value.

   (iii) Mating among unequal bits is productive which will result in a bit value equal to the difference in the mating bits.

   Following the above, the string $C$ for the example strings $A$ and $B$ will be

   $1 1 1 1 0 1$

   Therefore, the fitness of the “off-spring,” i.e., string $C$ is the sum of all the non-zero elements which is equal to five.

**Mating of First Generation Strings**

For example, consider the links 2 and 3 of the chain, Fig. 2. The first generation binary string of link-2 (from the matrix-2) is

$$1 0 1 0 0 0$$

and the first generation binary string of link-3 is

$$0 1 0 1 1 0$$

Then the mating of the links 2 and 3 yields an “off-spring” of fitness 5 represented by the binary string $1 1 1 1 1 0$.

In this manner, mating of each link with the other links one-by-one can be considered and the fitness of the resulting “off-spring” can be presented in the form of a matrix $F_1$. For example, for the six-link chain, Fig. 2, $F_1$ matrix will be
Second Generation Strings and Their Mating

Second generation strings are developed in a manner similar to that of the first generation but the elements 1 in the adjacency matrix \( A_2 \) correspond to the links separated by only one link and all the other elements will be zero. For example, consider the six-link Stephenson chain, Fig. 2, with respect to link-1, only links 3 and 5 are separated by one link. Hence only the elements corresponding to links 3 and 5 in the first row of the adjacency matrix \( A_2 \) will be 1, all other elements being zero. Similarly, the links 4, 5, and 6 are separated by one link from the link-2. With this understanding, the matrix \( A_2 \) for the above chain is

\[
A_2 = \begin{pmatrix}
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0
\end{pmatrix}
\]

(7)

Each row of the above matrix is considered as the secondary binary string of the concerned link, e.g., for link-5, the binary string is 1 1 0 1 0 0 and hence its secondary fitness is three.

The secondary binary strings can be mated following the rules (3) stipulated earlier which in turn yields the secondary fitness matrix, \( F_2 \).

For the Stephenson chain \( F_2 \) is

\[
F_2 = \begin{pmatrix}
0 & 3 & 4 & 3 & 5 & 3 \\
3 & 0 & 3 & 2 & 4 & 4 \\
4 & 3 & 0 & 3 & 3 & 5 \\
3 & 2 & 3 & 0 & 4 & 4 \\
5 & 4 & 3 & 4 & 0 & 2 \\
3 & 4 & 5 & 4 & 2 & 0
\end{pmatrix}
\]

(8)

Following the lines of first generation, a chain string for second generation can be written for every chain. For the Stephenson chain, it is


(9)

Now, working out on the above lines we get the chain string for the Watt chain, Fig. 3, as

\[6[16−3(4),2(2)] \]

(10)

Link wise, third generation, fourth generation strings, etc., can be generated for every chain. For example two links in a ten-link single d.o.f. chain are separated maximum by four links and it may be necessary to develop strings up to third generation.

Test For Isomorphism. Test for isomorphism among kinematic chains consists of comparing the chain strings generation wise, i.e., first, second and third generation, etc., strings of a chain need be compared respectively with the first, second, and third generation strings of the other chain. If they are identical, the chains are isomorphic, otherwise distinct. For example, comparison of the strings of both the Watt and Stephenson chains reveals that the first generation string itself is adequate to show that both the chains are distinct but the comparison of all possible (first, second, ...) generation strings constitutes the complete test satisfying both the necessary and sufficient requirements, thus making this method unique.

Distinct Inversions. A kinematic chain has as many inversions as the number of links in it. However, some of them may be identical. In order to get distinct inversions, it is only needed to compare the family fitness strings of different links of a chain. Usually, first generations strings are adequate but the necessary and sufficient strings will be of first and second generation for eight link chains and so on as described in the earlier sections. If the family fitness strings of two links are identical, the resulting inversions are identical, otherwise distinct.

For the six-link Stephenson chain, it is easy to see that there are three distinct inversions, i.e., when links (1 3), (2 4) and (5 6) are fixed. On the other hand, the six-link Watt chain has only two distinct inversions, i.e., when links (1 4) and (2 3 4 5) are fixed.

Best Chains and Inversions. It may be recalled that the fitness of a link is given by the number of nonzero elements in a
binary string. The fitness is in fact related to the number of design parameters such as link invariants (lengths, angles). Thus the first or second generation strings, etc., of a link correspond to the fitness of a link, i.e., greater fitness indicates involvement of more design parameters in generating the motion. Thus it can be noted that greater the family fitness of a link, its contribution to the motion generated by a chain is greater. For example, a chain with a greater fitness, i.e., sum of all the elements in the $F$ matrix will generate the specified motion such as function generation with greater accuracy or with less structural error. Inversion results when one of the links of a chain is grounded (fixed). It is obvious that it is better to fix a link with least fitness, so that more number of design parameters will be left to take part in motion generation. For example, link 2 or 4 in the six-link Stephenson chain, Fig. 2 has the least fitness as indicated by the sum of the elements in row 2 or 4 of the matrix $F$. Hence the inversion obtained by fixing one of these links will be the best inversion of this particular chain. Similar reasoning reveals that the inversion obtained by fixing one of the links 5 or 6 will lead to next best inversion. The third inversion obtained by fixing the links 1 or 3 will not be efficient from the motion generation point of view.

The above chain and the other examples not worked out here indicate that in general (i) links with least connectivity are preferable for grounding, (ii) out of many such links, the sum of the connectivities of all their immediately adjacent links matter, i.e., greater this sum better will be the link for fixing, (iii) if first generation strings and hence the fitness of two links are equal, compare the second generation strings and their fitness values to find distinct inversions and to rate them. The procedure may be extended if necessary to strings of successive generations.

Not withstanding, the above connectivity of the ground link depends upon the number of input motors and the output link (in case of function generator). In the latter case, input and output links should be far removed from one another. This may necessitate the selection of a higher connectivity link.

**Best Input Link.** It has been stated in the previous section that it is better to fix a link which has least family fitness value obtained from the fitness matrices. The same logic when extended to the selection of the input link suggests that a link with a maximum or high fitness value will make a good input link. Thus, in the six-link Stephenson chain, link 1 or 3 will be the best choice as the input link. The next best choice will be either link 5 or 6. In general higher connectivity links are preferable as input links.

**Comparison of Chains.** Let us consider two eight-link chains with different link-assortments, (i) four binary and four ternary links and (ii) six binary and two quaternary links.

The adjacency matrix and the corresponding fitness matrix can be written for each chain. Some of all the elements of the $F$ matrix is the fitness value of the chain, generation wise, higher value indicating greater ability of the chain to generate motion i.e. more accurately. The first generation fitness values of the chains consisting of higher connectivity links will invariably be less irrespective of the chain topology and hence such chains are inferior.

**Conclusion**

This method presents necessary and sufficient conditions for testing isomorphism uniquely.

The genetic approach presented here to detect isomorphism resembles by coincidence the Hamming number technique only up to first generation. But the concept of fitness and successive generation introduced in this paper enables the selection of best chain, best inversion and best input links and makes the test for isomorphism unique.

In order to compare the chains (Figs. 2 and 3), their dimensional synthesis is carried out by the same method using displacement equations developed in [7] for generating the same function with the same range, starting angles, etc. Details are not given here for want to space. However, sum of the structural errors squared is found to be less in case of Stephenson chain validating the theory presented.

Computation is extremely simple. From the motion generation point of view, chains consisting of links with higher connectivity appear to be inferior. Fixing a link with least connectivity will lead to a better inversion and if there are many such links, the one with high connectivity links as adjacent links will be preferable. Links with higher connectivity are preferable as input links. The method is unique as stated earlier, nevertheless, all the 230 distinct ten-link single d.o.f. chains are tested for confirmation.

**Appendix**

In general, the chain totals and row totals of every chain are more than adequate except for the special cases, i.e., chains with almost identical topology; if chain totals and row totals of one chain are one-to-one identical to those of the other chain, then the two chains are said to be isomorphic. This saves a lot of effort since the totals are obtained directly from the adjacency matrices by using the formulas given below.

Sum of the fitness values of $i$th row

\[ n = \text{number of links in the chain} \]

\[ C_n = \text{Sum of the adjacencies of all the links (generation wise)} \]

\[ V_i = \text{Sum of the vertex values of the links directly adjacent to link-}\]

\[ \text{link-}i \text{. Vertex value of a link is one less than its adjacency value.} \]

For illustration, consider the second row of the matrix (6).

\[ n = 6, \ C_2 = 2, \ C_6 = 14 \text{ and } V_2 = (2+2) = 4 \]

Therefore, sum of the elements of the second row of fitness matrix ($F_2$) = $(4\times 2) + 14 - (2\times 4) = 14$.

This may be verified from the matrix (7). Similarly, consider the second adjacency matrix (9) and consider its fourth row. Then, $n = 6, \ C_4 = 3, \ C_6 = 16 \text{ and } V_4 = 6$.

Therefore, sum of the elements of the fourth row of the fitness matrix (10) is $(4\times 3) + 16 - (2\times 6) = 16$.

This may be verified for the fourth row of the matrix (10).

The fitness value of the chain, generation-wise, may be obtained by summing up all the row values obtained as above. Also, it may directly be determined by using the expression

\[ 2 \sum_{i=1}^{n} (n-1)C_i - 2 \sum_{i=1}^{n} (C_i - 1) \]

**References**


